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Mats Carlin & Tom Kavli
University of Oslo & SINTEF Electronics & Cybernetics
Box 124 Blindern, N-0314 Oslo, NORWAY
Email : Mats.Carlin@ecy.sintef.no, Tom.Kavli@ecy.sintef.no
http://www.ifi.uio.no/ ~matsc

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Extruded aluminium sections may be characterised by their shape and some other production parameters. There is a need for finding resembling sections in order to model the extrusion process. Analysis of shape similarity is in nature very difficult. It is dependent on the perception, representation and context of a given shape. Interpretation of the similarity measures is of vital importance as we may want to build models of production parameters based on the shape features. This paper aims at performing a background analysis of the problem of similarity analysis of extruded aluminium section shapes.

Keywords : Shape, similarity, extrusion, aluminium, application

1 [illegible]

This paper describes a novel application of shape similarity within the aluminium extrusion industry. Several geometric shape properties are known to influence the production of aluminium sections. In this context we are interested in searching for visually and productionally similar shapes. The purpose of this paper is to discuss which problems occur and which methods are available to solve this particular problem. The paper provides a platform for later more detailed studies of different solution strategies and/or methods.

The paper is organised as follows. First we briefly introduce the reader to the aluminium extrusion process and sketch some applications of shape similarity. Next we define shape similarity and invariance mathematically. Then we discuss similarity from an application viewpoint and stress the importance of interpretation, context and some other aspects of interest. Some available techniques are presented on a system level.

2 [illegible]

Aluminium extrusion is a process where solid aluminium bars called billets are preheated to a temperature of 450-500 degrees Celsius and then pressed through a die (extrusion tool). The die has a three-dimensional geometry that forms the aluminium in a very similar way to squeezing toothpaste out of tooth paste tube. The resulting product is an aluminium extrusion with a two-dimensional cross section shape formed by the geometry of the exit of the die. At the exit of the die the aluminium may have a temperature of more than 500 degrees Celsius, and the sections may be quenched or cooled with pressured air, water spray or a bath of water. The aluminium extrusion is then guided down the cooling table by a puller and is cut in lengths in a semi-continuous process. At last the sections are sheared, cut to shorter lengths and packed. Some alloys may be heat treated as well. The most important quality features of aluminium extrusions is their mechanical properties and surface finish. The speed of the extrusion process depends on alloy, some press parameters and the complexity of the section in the die opening. At present only limited understanding and simple models exist on using the shape of the extruded section in modelling press parameters such as exit temperature and press speed.

One press at an extrusion plant may produce several thousands geometrically different sections every year. Up to a thousand of these may be new sections not previously produced. There is an immense need for models that may identify geometrical and productional similarities between new sections and previously produced sections. Such models could be used for several purposes; modelling press parameters, identifying good die designs, avoid scrap etc. Exact representations of each section exist as CAD drawings. There is a great variety in the section shapes. The most complex ones may have close to hundred different details and elements.

Our goal in this article is to study shape similarities of the section shapes from a general view point. We will assume a parametric description of the contours and we will only work on two-dimensional objects. Analysis of shape similarity is in nature different from other aspects of shape analysis, such as shape classification. Each new section shape forms a new class that may resemble some of the other classes (sections).

Aluminium extrusions are used for a large variation of applications, from elements for building and transport industry to cooling systems for electronics. Figure 1 shows a selection of different extrusion shapes.

[illegible]

The mathematical notion of shape similarity is invariance under a certain type of transformation, called a similarity transformation. This includes invariance to translation, rotation, reflection and scale (see figure 2).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} c \\ \pm c \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{1}$$

The term of similarity dates back to the Greek civilisation 2300 years ago. The Greek mathematician Euclid assembled most of that periods knowledge of mathematics in his astonishing 13 volume work *the Elements*. The Elements was reassembled from the different surviving pieces and fragments a century ago. Euclid covers triangle and polygon similarity in his sixth book, proposition 19 and 20 [1]. However Euclid's notion of similarity lacked the invariance to scale and reflection such as we use it today. A Euclidean transformation therefore includes translation and rotation, but not scale and reflection.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2}$$

A more general form of the similarity transformation is the affine transformation which in addition allows stretching and shearing of the geometrical object.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{3}$$

However similarity is above used in its strict sense, the shape should be perfectly similar to be called a similarity. A triangle may only be compared with another triangle, a quadrilateral with another quadrilateral and in the more general case an n-gon with an n-gon. The smallest deviation corrupts the similarity. For polygons or other parametrisations of a contour this an impractical way of using the notion of similarity, resemblance is a much wider concept, since we now may ask the questions: How much does a triangle resemble a quadrilateral? How similar is an n-gon to an m-gon ($n \neq m$)? This is the same question as we may ask in our present application: How much does one section resemble another section? Even worse: In what ways are one section similar to another?

Our goal is to represent a shape by a set of geometrically invariant parameters or attributes that are unique for the given shape. Invariance for rotation, reflection and translation is a minimum requirement, i.e. invariance under similarity transformation. We also want the selected invariant parameters to span the space of possible parametrisations as well as possible. In some sense the parameters should be 'orthogonal in the space of parametrisations'.

[illegible]

Similarity of shapes can be based on our perception of shape, in which case it will differ from person to person and from application to application. *Visual perception* of shape may differ a lot from *machine perception* of shape, i.e. algorithmic perception [2]. One example that may explain this is that a small curvature deviation from a straight line is visually much more obvious to us as humans than a small deviation of curvature on a curved line. For an algorithm this must be accounted for if we really want to mimic human perception. If we wish to assess the visual similarity between a number of different shapes, this proves to be a very difficult task as visual similarity is dependent on the ability of a person to find patterns and symmetries in the current objects. Psychological surveys on shape similarity show that the concept of similarity is tightly connected to a persons background, culture, experience, situation, expectations, mood, age and so on. However in some sense there is a common kernel of ways to find similarities based on the physiology of our perception. Symmetries, repeating patterns, structure and smoothness of the object influence how we consider an object.

Interpretation of the shape attributes is very important for the above described application. If one is to model a physical process, shape features which do have a statistical and physical interpretation such as area, perimeter, length, width, variation, skewness is much more attractive than shape features which do not have an interpretation, simply because the feature then adds to our understanding of the underlying model or problem.

The context and application of similarity must also be considered. In various contexts different objects may have different *contextual similarity.* Aluminium extrusions used in the building industry are contextually different from those used as cooling elements for the electronics industry. The variation within each class of objects may be much larger than the difference between two classes. The intended use of the section or the customer may induce rules of similarity between different shapes that are not purely based on shape, but also takes other parameters into account.

When a product with a certain shape is produced we may have *productional similarity* defined by some production parameters such as gross production, production rate, scrap causes, yield, economy etc. There may be limiting factors such as overall geometric attributes or certain details in the shape which to a large degree influences the production of the required shape. One way to assess productional similarity is to do a correlation analysis of the geometrical features with some key production parameter as the target. However, we should remember a correlation analysis only finds linear dependencies in the data and not non-linear dependencies.

We may also identify *representational similarity* by observing that some contours may be represented by a certain class of geometric parametrisations and therefore become similar or dissimilar. A simple illustration is the following example. A circle is difficult to represent correctly by a polygon with a limited number of vertices, while both rectangles and triangles are easily represented by polygons. In some sense the triangle and rectangle are closer to each other in representation than a circle regardless of the actual size and shape of each of these objects. If we represent the same objects by their skeleton a completely different result may occur. Representations may be based on the contour of the object, the interior (structure) of the object (medial axis transform) or some transformation of the object (Fourier transform).

The next important aspect of shape similarity is shape *symmetry.* How do we measure and/or detect symmetry of an object? We have different kinds of symmetry, such as mirror symmetry, rotational symmetry or radial symmetry. Symmetry can be either a binary feature (yes/no), or an integral feature (where n is the number of axes of symmetry) or even a continuous feature (including almost symmetrical shapes) [3].

We may also consider shape *complexity.* Our notion of complexity may differ based on the representation we have. Complexity of a boundary is quite different from structural or topological complexity.. Some suggested methods for boundary complexity include fractal dimensions, curvature measures and entropy measures.

The concept of *structure* is also important for our understanding of similarity. The overall structure or topology is based on the number of holes and their individual ordering and the number of protrusion and deficiencies. It is important to distinguish between overall structure and the structure of details, hence scale influences our understanding of structural similarity. In some cases we are only interested in the details (see figure 3).

[illegible]

As representation is crucial for shape similarity analysis, we must discuss the shape representation. In our case each contour is originally represented by line segments, circle arches and splines in a CAD system. Line segments, circle arches and splines are all parametric and could hence be used directly in our computation of shape features. However in many cases we may simplify our analysis if we allow simple approximations to the original drawing. The simplest approximation of a continuous contour is by a polygon, which consists of a finite number of line segments each represented by its end points or vertices. Each inner and outer contour is resampled by a polygon with a chosen predefined maximum deviation from the contour. Polygons can be viewed as first-order B-splines with the vertices as control points. It is easy to generalise the polygons by higher-order B-splines forming more continuous representations. By using B-splines it is easy to control the maximum deviation from the original contour and if we need higher resolution it is easy to resample the original contours.

B-splines have the following nice properties [4] :

- Convex hull property
- Linear precision
- Affine invariance
- Symmetry
- Endpoint interpolation
- Variation diminishing property
- Local control

The parametric contours are boundary representations of the object. However for certain parts of the analysis it may be an enhancement to transform the contour representation to an interior representation, such as skeletons (through the medial axis transform [5]). The skeleton or medial axis of a plane object can be viewed as those interior points of the object which have equal distance to at least two distinct nearest points on the contour. The points of the skeleton are the loci of inscribed circles touching the contour at two or more points. The simplest method to compute the skeleton of a polygon is through a generalised Voronoi diagram. The Voronoi diagram of a set of distinct points in the plane consists of those points dividing the plane into Voronoi regions [6]. A Voronoi region contains all points closer to one point in the original set than the others. Hence the boundary between each Voronoi region is the Voronoi diagram. Any point on the Voronoi diagram has equal distance to at least two nearest points of the original point set. It is possible to generalise the Voronoi diagram to more general geometric elements than points, such as line segments. Then it is relatively easy to compute the generalised Voronoi diagram of a polygon that contains line segments (the edges of the polygon) and points (the vertices of the polygon) as geometric elements. The medial axis is fully contained within the generalised Voronoi diagram and can easily be found by deleting certain edges of the generalised Voronoi diagram [7]. In addition to the internal skeleton, it is possible to compute external skeletons for holes and deficiencies in the object. Figure 4 shows the external and internal skeletons of one sample section.

[illegible]

Any shape feature should have some physical, statistical or structural interpretation. Interpretations are important for a process like aluminium extrusion, since interpretations may provide understanding and knowledge about the process. In addition some abstract features such as symmetry and complexity are important. In this article we will discuss a number of shape features. Shape features are in general some values extracted algorithmically from a shape representation.

[illegible]

A number of general shape features do have a physical and statistical interpretation, such as size, area, length, width, perimeter or moments of inertia. Some of the above mentioned features may be generalised using techniques involving geometrical moments. Both area moments and contour moments exist.

Area moments of order p+q are defined as

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y) dx dy \tag{4}$$

for a two-dimensional piecewise continuous function $f(x,y)$ [8]. A uniqueness theorem exists, but moments of higher order than 3 are seldom used due to the fact that they are sensitive to sampling and digitisation errors in two-dimensional images. However for our application we do have an exact parametric representation of the contours, and higher order moments should be possible to use. The zeroth order moment is simply the area or mass of the object, first order moments are indicating the mass center and second order moments are the moments of inertia (and may be used to identify principal axes, image ellipse and radii of gyration) [9]. Statistically first order moments denote mean values, second order moments covariance, third order moments skewness and fourth order moments kurtosis. In our case the moments will be interpreted as mass balance parameters for the aluminium flow in the extrusion tool. We will not here go into details on the moment theory, but only state that it is possible to construct orthogonal moments and invariant moments from pure geometrical area moments.

Contour moments are defined as [10]

$$l_{pq} = \oint_c x^p y^q dl, \quad \forall x, y \in L \tag{5}$$

The zeroth order contour moment is simply the perimeter length, first order contour moments indicate the centroid, second order contour moments may be used to identify the principal axis etc.

[illegible]

Invariant features may be obtained by circumscribing the object by some predefined geometrical figure, like a circle or a rectangle. The circumscribing figure is here defined as the smallest enclosing figure of a certain class that covers the object. Some possible classes of figures are triangles, quadrilaterals, polygons (n-gons), circles or ellipses. In general we may impose some additional restrictions on the circumscribing figures such as number of parameters (edges), convexity and regularity.

The basic operation needed for convex enclosures is to compute the convex hull, i.e. the smallest convex set covering the complete object. All other convex enclosures such as circles, rectangles, convex n-gons etc. can be computed from the convex hull. Invariant features for the convex hull include perimeter length, diameter, ratio of areas or perimeters. For other circumscribing figures similar features may be computed. Some examples are given below.

A typical circle feature is the diameter of the circumscribing circle. Circleness may be defined as ratio of the area of the object over the area of the circle. Similar features for the circumscribing rectangle are length and width of the rectangle. Rectangleness may be defined as the ratio of the area of the object over the area of the circumscribing rectangle. It is easy to see that these features are similarity invariant. For objects with holes another circumscribing figure is the outer contour.

All these features may be said to have a physical interpretation. In our case many of them make sense, since there are maximum values for circumscribing circles and rectangles based on the dimensions of the die.

Feature	Area	Perimeter
Triangleness	$\frac{2A_{object}}{length \cdot width}$	$\frac{P_{object}}{P_{triangle}}$
Rectangleness	$\frac{A_{object}}{length \cdot width}$	$\frac{P_{object}}{2 \cdot length \cdot width}$
Fill factor	$\frac{A_{object}}{A_{outercontour}}$	$\frac{P_{object}}{P_{outercontour}}$
Compactness	$\frac{A_{object}}{A_{convexhull}}$	$\frac{P_{object}}{P_{convexhull}}$
Circleness	$\frac{A_{object}}{\pi(radius)^2}$	$\frac{P_{object}}{2\pi(radius)}$

Table 1 Features based on circumscribing figures

[illegible]

Elliptic Fourier descriptors are based on the fact that a parametric description of the contours as $x(t)$ and $y(t)$ may be expressed as an elliptic Fourier expansion in matrix form

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ d_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos(kt) \\ \sin(kt) \end{bmatrix} \tag{6}$$

where a_0 and d_0 denote the center of the object and each a_k, b_k, c_k and d_k are the parameters of the k'th image ellipse [11].

To create similarity invariants one may use different matrix trace measures for each k'th matrix. These may be interpreted as parameters of each k'th image ellipse, such as semi-axis length, area, location and relative orientation of each image ellipse [11]. In reality these can be viewed as elliptic spectral features of the contour and the distribution could reveal the complexity of the contour. Application to symmetry detection are given in [12]. There is a connection between Elliptic Fourier descriptors and geometric moments through the Fourier-Mellin transform [13]. However there is currently no obvious way of combining inner and outer contours in a common framework using this method.

[illegible]

In our analysis of aluminium extrusions we are interested in two different types of structural primitives. A priori we know that some details limit the production of the extrusions. Such details could be sharp angles or certain details with a large variation in thickness. The second type of primitive of interest is overall structure, to enable us to classify the overall structure of the extrusion. Structural features may be extracted from either skeletons or contours.

Typical low level primitives of a skeleton are end points, break points, branch points, protrusions and loops. For contours we may identify low level primitives such as number of holes, protrusions (convexities) and deficiencies (concavities). These can be combined to higher order primitives using a set of rules, i.e. a grammar [14]. Hence, we may say that structure is in essence semantics. Typical top-level structural primitives may be U-shape, L-shape, I-shape, H-shape, comb-shape, C-shape etc. However strict rules may work rather poorly, as small deviations in shape can cause a skeleton or a contour representation to change a lot structurally. It seems better to apply fuzzy rules. One of the main problems of this approach is the large variation in the structure of the extruded aluminium sections. We may ask if it is possible at all to build a complete shape grammar, which embraces all the structural shapes of our data? Rules should be based on a theory, not be ad hoc.

Another road to structural analysis is to define a number of basic primitives such as a circle, ellipse, triangle, rectangle, pentagon or even a specific primitive for the searched detail such as 'open screw hole', 'closed screw hole' or 'cooling fin type B'. Then we may perform a direct matching of the object or part of the object with respect to the primitive using a set theoretic distance measures like the Hausdorff distance. The Hausdorff distance is defined as the maximum deviation between two sets when aligned [15]

$H(A, B) = \max(h(A, B), h(B, A))$

where

$h(a, b) = \max_{a \in A} \min_{b \in B} ||a - b||$.

Other such set theoretic measures such as the template distance or transport distance are given in [2].

[illegible]

There are many ways of defining measures of complexity. Complexity is related to the smoothness of the object contours. Smoothness is defined by curvature and the spectral distribution of the contour is one possible measure. Another measure of complexity is the fractal dimension of the contour, a third is contour entropy. The angular entropy between two different line segments may be defined as

$H_\alpha = \log_2(2\pi/\alpha)$

where α is the angle between the two line segments. For a piecewise linear representation we may sum over all knots. However in some sense we should also include the number of knots and the distance between each knot, i.e. the length of each line segment.

For skeletons similar measures may be developed based on the branches of the skeleton with the number of knots, length of each branch and the angle between two branches at a knot.

[illegible]

In this paper we have performed a background analysis of the problem of similarity analysis of extruded aluminium section shapes. This problem has as far as we are aware of not been reported in the literature. Extruded aluminium sections may be characterised by their shape and some other production parameters. A description of the extrusion process is given to provide background information about the problem. Then follows a discussion on the difficult problem of shape similarity and which perceptional, interpretational, contextual and representational difficulties arise for our problem. We have used shape features such as

- Moments
- Features based on circumscribing figures
- Elliptic Fourier descriptors
- Structural features
- Complexity features

Interpretation of the similarity measures is of vital importance as we may want to build models of production parameters based on the shape features.

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